Exponents and Logarithms

Exam Questions
Multiple Choice

1. If \( \log_2 x = 4 \), then \( \log_2 (2x) \) is equal to:

   a. 5  
   b. 8  
   c. 16  
   d. 32

2. Identify the value of the \( x \)-intercept of the function \( y = \ln(x - 2) \).

   a. -1  
   b. 0  
   c. 2  
   d. 3

3. Which equation is represented by the graph sketched below.

   a. \( y = \left(\frac{1}{2}\right)^{-x} \)
   b. \( y = \left(\frac{1}{2}\right)^{x} \)
   c. \( y = 2^{x} \)
   d. \( y = -2^{x} \)
4. The graph of \( y = \log_2(2x + 6) \) intersects the graph of \( y = 4 \) at:

- a. \( x = -1 \)
- b. \( x = 1 \)
- c. \( x = 5 \)
- d. \( x = 14 \)

5. The graph of \( y = \left(\frac{1}{2}\right)^x \) compared to the graph of \( x = \left(\frac{1}{2}\right)^y \) is a:

- a. reflection in the \( x \)-axis
- b. reflection in the \( y \)-axis
- c. reflection in the line \( y = x \)
- d. reciprocal function

6. The graph of the function \( f(x) \) shown below is best described by the equation:

- a. \( f(x) = 2^{x+3} \)
- b. \( f(x) = 2^x + 3 \)
- c. \( f(x) = 2^{x-3} \)
- d. \( f(x) = 2^x - 3 \)
7. Which of the following is a reasonable estimate for the value of $\log 350$?

a. 2  b. 2.5  c. 2.8  d. 3

8. Solve: $e^{\ln(5-x)} = 7$

a. -2  b. $-\ln 2$  c. $\ln 7 - \ln 5$  d. $\frac{7}{5}$

9. Simplify the following expression: $\frac{1}{2} \log_a 36 - \log_a 2$

a. $\log_a 3$  b. $\log_a 4$  c. $\log_a 9$  d. $\log_a 12$

10. Which of the following is closest to the value of $\log_2 40 + \log_5 125$?

a. 3  b. 8  c. 10  d. 45
11. The \( x \)-intercept of the graph of \( y = 3^x - 1 \) is:

a. -1 \quad b. 0 \quad c. 1 \quad d. 2

12. The expression \( 2 \log x - \frac{1}{3} \log y \) as a single logarithm is:

\[ \log \frac{x^2}{\sqrt[3]{y^3}} \]

a. \( \log \frac{x^2}{\sqrt[3]{y^3}} \) \quad b. \( \log \frac{2x}{3y} \) \quad c. \( -\log x^2 \sqrt[3]{y} \) \quad d. \( \log \left(x^2 - \sqrt[3]{y^3}\right) \)

13. Determine the value of \( \log_9 \left( \log_3 27 \right) \).

a. \( \frac{1}{3} \) \quad b. \( \frac{1}{2} \) \quad c. 2 \quad d. 3

14. Identify an equivalent expression for \( 1 + \log_2 5 \).

a. \( \log_2 5 \) \quad b. \( \log_2 7 \) \quad c. \( \log_2 10 \) \quad d. \( \log_2 11 \)
15. Solve: \( 7^{\log_7 2} = x \)

a. \( x = 1 \)  
   b. \( x = 2 \)  
   c. \( x = 7 \)  
   d. \( x = 49 \)

16. Identify the logarithmic form of \( 5^x = 6 \).

a. \( \log_5 x = 6 \)  
   b. \( \log_5 6 = x \)  
   c. \( \log_6 x = 5 \)  
   d. \( \log_6 5 = x \)
Written Response

17. Given \( \log_b a = 3 \), give one example of possible values of \( a \) and \( b \) that make this equation true. (1 mark)

**Solution**

Answers will vary but \( b^3 = a \).

Some possible solutions are: \( a = 8, b = 2 \)  

or  

\( a = 27, b = 3 \)  

or  

\( a = 64, b = 4 \)

18. Frank tried to expand a logarithmic expression using the laws of logarithms. He made one error.

*Frank’s solution:*  

\[ \log_a \left( \frac{x + 2}{zw} \right) = \log_a x + \log_a 2 - \log_a z - \log_a w \]

Write the correct solution. (1 mark)

**Solution**

Correct solution:  

\[ \log_a \left( \frac{x + 2}{zw} \right) = \log_a (x + 2) - \log_a z - \log_a w \]

1 mark for correct solution

19. Estimate the value of \( \log_5 35 \).

Justify your answer. (1 mark)

**Solution**

\[ 5^2 = 25 \quad \quad \quad \quad 5^3 = 125 \]

The value of \( \log_5 35 \) is more than 2 but less than 2.5.  

\( \frac{1}{2} \) mark for justification

\( \frac{1}{2} \) mark for estimated answer
20. Claire correctly solves the following equation:

\[ \log_2(6-x) + \log_2(3-x) = 2. \]

She finds two possible values of \( x \): \( x = 2 \) and \( x = 7 \).
Identify which one of these values is unacceptable and explain why. (1 mark)

**Solution**

If \( x \) is greater than 3, you have a negative argument \( \therefore x = 2 \) but \( x \neq 7 \).

or

The domain is restricted to values of \( x < 3 \) \( \therefore x = 2 \).

1 mark for explanation

21. Which expression has a larger value?

\( \log_2 36 \) or \( \log_3 80 \)

Justify your answer. (1 mark)

**Solution**

**Method 1**

\[
\begin{align*}
\log_2 36 & = 2^\frac{2^5}{2} = 5.1 \\
\log_3 80 & = 3^\frac{3^3}{3^4} = 3.9
\end{align*}
\]

\( \therefore \log_2 36 \) is the larger value

1 mark for justification

22. Which of the following equations could be solved without the use of logarithms?

Without actually solving the problem, explain your choice. (1 mark)

\[ 4^x = 10^{3x+1} \]

or

\[ \left( \frac{1}{3} \right)^{2x+1} = 27^{4x-1} \]

**Solution**

\( \left( \frac{1}{3} \right)^{2x+1} = 27^{4x-7} \) can be solved without the use of logarithms because \( \frac{1}{3} \) and 27 can both be changed to a base of 3.

1 mark for explanation
23. Using the laws of logarithms, expand:  
\[ \log_x \left( \frac{x \cdot y}{z} \right) \]  
\[ \log_x x + \log_x y - \log_x z \]  
\[ 1 \text{ mark for product rule} \]  
\[ 1 \text{ mark for quotient rule} \]  
(2 marks)

24. Determine the \( x \)-intercept and \( y \)-intercept of \( y = \log_2 (x + 4) - 1 \).  
(2 marks)

Solution

Substitute \( x \) with 0.
\[ y = \log_2 4 - 1 \]
\[ y = 2 - 1 \]
\[ y = 1 \]
\[ \therefore y \text{-intercept is 1} \]

Substitute \( y \) with 0.
\[ 0 = \log_2 (x + 4) - 1 \]
\[ 1 = \log_2 (x + 4) \]
\[ 2 = x + 4 \]
\[ -2 = x \]
\[ \therefore x \text{-intercept is } -2 \]

25. Solve the following equations algebraically:  
\[ \log_3 (x - 4) + \log_3 (x - 2) = 1 \]  
(3 marks)

Solution

Method 1
\[ \log_3 (x - 4) + \log_3 (x - 2) = 1 \]
\[ \log_3 (x - 4)(x - 2) = 1 \]
\[ 3^1 = (x - 4)(x - 2) \]
\[ 3 = x^2 - 6x + 8 \]
\[ 0 = x^2 - 6x + 5 \]
\[ 0 = (x - 5)(x - 1) \]
\[ x = 5 \]
\[ x = 1 \]
\[ \frac{1}{2} \text{ mark for solving for } x \text{ within a quadratic equation} \]
\[ \frac{1}{2} \text{ mark for rejecting extraneous root} \]

3 marks
26. a. Sketch the graph of \( y = 3^x \).  

Solutions

\[ y = 3^x \]

\[ (1, 3) \]

\[ (0, 1) \]

b. Explain how the graph of \( y = 3^x \) can be used to sketch the graph of \( y = \log_3 x \).

(1 mark)

To graph \( y = \log_3 x \), you can reflect the graph of \( y = 3^x \) over the line \( y = x \).

or

You can switch the \( x \) and \( y \) coordinates of \( y = 3^x \) to get the graph of \( y = \log_3 x \).

1 mark for explanation
27. Determine the value of $y$ in the following equation: 

$$\log_x 27 - \log_x 3 = 2 \log_x y$$

**Solution**

$$\log_x 27 - \log_x 3 = 2 \log_x y$$

$$\log_x \frac{27}{3} = 2 \log_x y$$

$$\log_x 9 = \log_x y^2$$

$$9 = y^2$$

$$y = \pm 3$$

$$y = 3$$

3 marks

28. The number of times a website is visited can be modeled by the function:

$$A = 800 (e)^{rt}$$

where $A =$ the total number of visitors at time $t$

$t =$ the time in days ($t \geq 0$)

$r =$ the rate of growth

After 5 days, 40 000 people have visited the site. Determine the number of visitors expected after 9 days. Express your answer as a whole number. (calculator) (3 marks)

**Solution**

**Method 1**

$$40000 = 800 e^{5r}$$

$$\frac{40000}{800} = e^{5r}$$

$$\ln 50 = \ln e^{5r}$$

$$\ln 50 = 5r$$

$$\ln 50 = r$$

$$r = 0.782404601$$

$$A = 800 e^{(0.782404601)(9)}$$

$$A = 914610.103$$

$$A = 914610$$

3 marks
29. a. Sketch the graph of \( y = \ln(x) \). (2 marks)

**Solutions**

a) 

\[
\begin{align*}
(1, 0) & \quad (e, 1)
\end{align*}
\]

- \( \frac{1}{2} \) mark for increasing logarithmic function
- \( \frac{1}{2} \) mark for \( x \)-intercept at \((1, 0)\)
- \( \frac{1}{2} \) mark for consistent point on logarithmic function
- \( \frac{1}{2} \) mark for vertical asymptotic behaviour

b. Sketch the graph of \( y = -\ln(x - 2) \). (2 marks)

b) 

\[
\begin{align*}
(3, 0) & \quad (e + 2, -1)
\end{align*}
\]

- 1 mark for reflection in \( x \)-axis
- 1 mark for horizontal shift

2 marks
30. Solve algebraically: \[ 10^{3x} = 7^{x+5} \]  
\[ \text{(calculator)} \]  
\[ (3 \text{ marks}) \]

**Solution**

**Method 1**

\[ 10^{3x} = 7^{x+5} \]

\[ \log_{10}^{3x} = \log_{7}^{x+5} \]  
\[ 3x \log_{10} 10 = (x + 5) \log_{7} 7 \]  
\[ 3x \log_{10} 10 = x \log_{7} 7 + 5 \log_{7} 7 \]

\[ 3x \log_{10} 10 - x \log_{7} 7 = 5 \log_{7} 7 \]

\[ x = \frac{5 \log_{7} 7}{3 \log_{10} 10 - \log_{7} 7} \]  
\[ x = 1.960 \, 873 \]  
\[ x = 1.961 \]  
\[ \text{(3 marks)} \]

31. Jess invests $12,000 at a rate of 4.75% compounded monthly. How long will it take for Jess to triple her investment? Express your answer in years, correct to 3 decimal places.  
\[ \text{(calculator)} \]  
\[ (3 \text{ marks}) \]

**Solution**

**Method 1**

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ 36,000 = 12,000 \left(1 + \frac{0.0475}{12}\right)^{12t} \]  
\[ 3 = \left(1 + \frac{0.0475}{12}\right)^{12t} \]

\[ \ln 3 = \ln \left(1 + \frac{0.0475}{12}\right)^{12t} \]  
\[ \ln 3 = 12t \ln \left(1 + \frac{0.0475}{12}\right) \]  
\[ t = \frac{\ln 3}{12 \ln \left(1 + \frac{0.0475}{12}\right)} \]

\[ t = 23.174 \, 425 \]  
\[ t = 23.174 \, \text{years} \]  
\[ \text{(3 marks)} \]
32. Solve the following equation:  

\[ 2 \log_4 x - \log_4 (x + 3) = 1 \]  

**Solution**

\[ 2 \log_4 x - \log_4 (x + 3) = 1 \]

\[ \log_4 \left( \frac{x^2}{x + 3} \right) = 1 \]  
1 mark for power rule  
1 mark for quotient rule  

\[ 4^1 = \left( \frac{x^2}{x + 3} \right) \]  
1 mark for exponential form  

\[ 4(x + 3) = x^2 \]  
\[ x^2 - 4x - 12 = 0 \]  
\[ (x - 6)(x + 2) = 0 \]  
\[ x = 6 \]  
\[ x \approx 2 \]  
\[ \frac{1}{2} \] mark for solving for \( x \)  
\[ \frac{1}{2} \] mark for rejecting extraneous root  

4 marks

33. An earthquake in Vancouver had a magnitude of 6.3 on the Richter scale. An earthquake in Japan had a magnitude of 8.9 on the Richter scale. How many times more intense was the Japan earthquake than the Vancouver earthquake? You may use the formula below:

\[ M = \log \left( \frac{A}{A_0} \right) \]

where \( M \) is the magnitude of the earthquake on the Richter scale  
\( A \) is the intensity of the earthquake  
\( A_0 \) is the intensity of a standard earthquake

Express your answer as a whole number.  

**Solution**

**Method 1**

**Vancouver:** substitute \( M = 6.3 \)

\[ 6.3 = \log \left( \frac{A}{A_0} \right) \]

\[ 10^{6.3} = \frac{A}{A_0} \]

\[ A = 10^{6.3} A_0 \]

**Japan:** substitute \( M = 8.9 \)

\[ 8.9 = \log \left( \frac{A}{A_0} \right) \]

\[ 10^{8.9} = \frac{A}{A_0} \]

\[ A = 10^{8.9} A_0 \]

To compare the two earthquakes divide their intensities.

\[ \frac{\text{the intensity of Japan}}{\text{the intensity of Vancouver}} = \frac{10^{8.9} A_0}{10^{6.3} A_0} = 10^{8.9 - 6.3} = 10^{2.6} \]

\[ \approx 398 \]  

1 mark for comparison  

2 marks
34. a. Sketch the graph of \( f(x) = 3^x + 1 \). (2 marks)

**Solution**

\[ a) \]

\[ (0, 2) \]

\[ (1, 4) \]

\[ \frac{1}{2} \text{ mark for increasing exponential function} \]
\[ \frac{1}{2} \text{ mark for } y\text{-intercept at } (0, 2) \]
\[ \frac{1}{2} \text{ mark for asymptote at } y = 1 \]
\[ \frac{1}{2} \text{ mark for consistent point on exponential function} \]

(2 marks)

b. Sketch the graph of \( f^{-1}(x) \). (1 mark)

\[ b) \]

\[ (2, 0) \]

\[ (4, 1) \]

1 mark for consistent graph of the inverse

1 mark
35. Given \( \log_a 9 = 1.129 \) and \( \log_a 4 = 0.712 \), find the value of \( \log_a 12 \). (3 marks)

**Solution**

**Method 1**

\[
\log_a 9 = 1.129 \\
\log_a 3^2 = 1.129 \\
2 \log_a 3 = 1.129 \\
\log_a 3 = 0.5645 \\
\log_a 12 = \log_a (4 \cdot 3) \\
= \log_a 4 + \log_a 3 \\
= 0.712 + 0.5645 \\
= 1.2765 \\
= 1.277 \\
\]

1 mark for power rule

1 mark for writing 12 as a product

1 mark for product rule

3 marks

36. Solve the following equation: (4 marks)

\[
2 \log_2 (x-1) - \log_2 (x-5) = \log_2 (x+1)
\]

**Method 3**

\[
\log_2 (x-1)^2 - \log_2 (x-5) - \log_2 (x+1) = 0 \\
\log_2 \frac{(x-1)^2}{(x-5)(x+1)} = 0 \\
2^0 = \frac{(x-1)^2}{(x-5)(x+1)} \\
x^2 - 4x - 5 = x^2 - 2x + 1 \\
-6 = 2x \\
\therefore \text{no solution} \\
\]

2 marks for logarithmic rules (1 mark for power rule, 1 mark for quotient rule)

1 mark for exponential form

\( \frac{1}{2} \) mark for solving for \( x \)

\( \frac{1}{2} \) mark for no solution

4 marks
37. Determine how many monthly investments of $50 would have to be deposited into a savings account that pays 3% annual interest, compounded monthly, for the account’s future value to be $50,000.

Use the formula: \( FV = \frac{R[(1 + i)^n - 1]}{i} \)

where: 
- \( FV \) = the future value
- \( R \) = the investment amount
- \( i \) = the annual interest rate
- \( n \) = the number of compounding periods per year

Express your answer as a whole number. (calculator) (3 marks)

**Solution**

\[
50 000 = \frac{50 \left( 1 + \frac{0.03}{12} \right)^n - 1}{0.03/12}
\]

\[
50 000 = \frac{50 \left(1 + 0.0025\right)^n - 1}{0.0025}
\]

\[
50 000 = 20 000 \left(1.0025^n - 1\right)
\]

\[
2.5 = 1.0025^n - 1
\]

\[
3.5 = 1.0025^n
\]

\[
\log 3.5 = \log 1.0025^n
\]

\[
\log 3.5 = n \log 1.0025
\]

\[
n = \frac{\log 3.5}{\log 1.0025}
\]

\[
n = 501.73
\]

: 502 monthly investments are needed. (3 marks)
38. A population of 500 bacteria will triple in 20 hours. (calculator)

Using the formula given below,

\[ A = Pe^{rt} \]

\( A \) = population after \( t \) hours  
\( P \) = initial population  
\( r \) = rate of growth  
\( t \) = time in hours

a. Determine the rate of growth \( r \). (2 marks)

\[
\begin{align*}
1500 &= 500e^{20r} \\
3 &= e^{20r} \\
\ln 3 &= \ln e^{20r} \\
\ln 3 &= 20r \cdot \ln e \\
r &= \frac{\ln 3}{20} \\
r &= 0.054930614 \\
\end{align*}
\]

b. Determine how many hours it will take for the initial population to double with the same rate of growth. (2 marks)

\[
\begin{align*}
1000 &= 500e^{0.054930614t} \\
2 &= e^{0.054930614t} \\
\ln 2 &= \ln e^{0.054930614t} \\
\ln 2 &= 0.054930614t \cdot \ln e \\
t &= \frac{\ln 2}{0.054930614} \\
t &= 12.619 \text{ hours}
\end{align*}
\]
39. Sketch the graphs of:

a. \( y = \left( \frac{1}{4} \right)^x \)  

\textbf{Solution}

\textbf{a)}

\[
\begin{array}{c}
\text{(1, 0.25)} \\
\text{(2, 0.0625)} \\
\text{(4, 0.016)}
\end{array}
\]

\text{1 mark for a vertical stretch by a factor of 2 of the graph consistent with a)}

b. \( y = 2 \left( \frac{1}{4} \right)^x \)  

\text{1 mark for a vertical stretch by a factor of 2 of the graph consistent with a)}
40. Evaluate: \[
\frac{1}{2} \log_3 144 - \log_3 4 + 2 \log_3 3
\]

**Solution**

\[
\log_3 (144)^{\frac{1}{2}} - \log_3 4 + \log_3 (3)^2
\]

\[
\log_3 12 - \log_3 4 + \log_3 9
\]

\[
\log_3 \left( \frac{12 \cdot 9}{4} \right)
\]

\[
\log_3 27
\]

\[
3
\]

1 mark for power rule

½ mark for product rule

½ mark for quotient rule

1 mark for evaluating a logarithm

3 marks

---

41. Solve the following equation:

\[
\log_4 (x + 2) + \log_4 3 = \log_4 x
\]

**Method 1**

\[
\log_4 (x + 2) + \log_4 3 = \log_4 x
\]

\[
\log_4 (x + 2)^3 = \log_4 x
\]

\[
3(x + 2) = x
\]

\[
3x + 6 = x
\]

\[
x = -3
\]

No solution

1 mark for product rule

1 mark for equating arguments

½ mark for solving for \(x\)

½ mark for rejecting extraneous root

3 marks

---

**Method 2**

\[
\log_4 (x + 2) + \log_4 3 = \log_4 x
\]

\[
\log_4 (x + 2) + \log_4 3 - \log_4 x = 0
\]

\[
\log_4 \left( \frac{3(x + 2)}{x} \right) = 0
\]

\[
4^0 = \frac{3x + 6}{x}
\]

\[
x = -3
\]

\[
\checkmark
\]

1 mark for logarithmic rules (½ mark for product rule; ½ mark for quotient rule)

1 mark for exponential form

½ mark for solving for \(x\)

½ mark for rejecting extraneous root

3 marks
42. Identify which of these values is greater. Justify your answer. 

\[ \log_5 80 \quad \log_5 30 \] 

**Solution**

\[ 5^2 = 25 \quad \log_5 80 \text{ is less than 3} \]
\[ 5^3 = 125 \]

\[ 3^2 = 27 \quad \log_3 30 \text{ is more than 3} \]
\[ 3^4 = 81 \]

\[ \therefore \log_3 30 \text{ is greater} \]

1 mark for justification

1 mark

43. Solve: 

\[ 2^{5x} = 3(5)^{x-3} \] 

(calculator) 

**Solution**

\[ \log 2^{5x} = \log \left[ 3(5)^{x-3} \right] \]
\[ 5x \log 2 = \log 3 + (x - 3) \log 5 \]
\[ 5x \log 2 = \log 3 + x \log 5 - 3 \log 5 \]
\[ 5x \log 2 - x \log 5 = \log 3 - 3 \log 5 \]
\[ x(5 \log 2 - \log 5) = \log 3 - 3 \log 5 \]
\[ x = \frac{\log 3 - 3 \log 5}{5 \log 2 - \log 5} \]
\[ x = -2.009 \]

½ mark for applying logarithms

1 mark for product rule

1 mark for power rule

½ mark for collecting like terms

½ mark for isolating x

½ mark for evaluating a quotient of logarithms

4 marks
44. A lake affected by acid rain has a pH of 4.4. 
A person suffering from heartburn has a stomach acid of pH of 1.2. 
The pH of a solution is defined as \( \text{pH} = -\log[H^+] \) where \([H^+]\) is the hydrogen ion concentration. 
How many times greater is the hydrogen ion concentration of the stomach than that of the lake? Express your answer as a whole number. \( \text{calculator} \) \( 2 \text{ marks} \)

\[
\begin{align*}
\text{Lake} & : 4.4 = -\log[H^+] \\
& : -4.4 = \log[H^+] \\
& : 10^{-4.4} = [H^+] \\
\text{Stomach} & : 1.2 = -\log[H^+] \\
& : -1.2 = \log[H^+] \\
& : 10^{-1.2} = [H^+] \\
\end{align*}
\]

\[ \frac{[H^+]_{\text{stomach}}}{[H^+]_{\text{lake}}} = \frac{10^{-1.2}}{10^{-4.4}} = 10^{3.2} = 1585 \]

45. Solve: \( 2\log_4 x - \log_4 (x + 3) = 1 \) \( 4 \text{ marks} \)

**Solution**

\[
\begin{align*}
2\log_4 x - \log_4 (x + 3) & = 1 \\
\log_4 \left( \frac{x^2}{x + 3} \right) & = 1 \\
4^1 & = \frac{x^2}{x + 3} \\
4x + 12 & = x^2 \\
x^2 - 4x - 12 & = 0 \\
(x - 6)(x + 2) & = 0 \\
x & = 6 \\
\end{align*}
\]

\( \frac{1}{2} \text{ mark for solving for } x \) \( \frac{1}{2} \text{ mark for rejecting extraneous root} \) \( 4 \text{ marks} \)
46. a. Sketch the graph of \( f(x) = \log_5(x - 1) \). (2 marks)

Solution

a) ![Graph of \( f(x) = \log_5(x - 1) \) with points and asymptote marked.]

\( \frac{1}{2} \) mark for vertical asymptote at \( x = 1 \)
\( \frac{1}{2} \) mark for \( x \)-intercept at \( x = 2 \)
\( \frac{1}{2} \) mark for increasing logarithmic function
\( \frac{1}{2} \) mark for consistent point on the logarithmic graph

2 marks

b. Sketch the graph of \( f^{-1}(x) \). (1 mark)

b) ![Graph of the inverse function \( f^{-1}(x) \) with point and domain marked.]

1 mark for the graph of the inverse function consistent with a)
47. Kim solved the following logarithmic equation:

\[
\log_2 \left( \frac{-x}{3} \right) = \log_2 (x - 4)
\]

\[
-\frac{x}{3} = x - 4
\]

\[
-x = 3x - 12
\]

\[
-4x = -12
\]

\[
x = 3
\]

Explain why \(x = 3\) is an extraneous root. (1 mark)

**Solution**

\(x = 3\) is an extraneous solution because the argument in a logarithmic equation cannot be negative.

48. Solve: \(6(5)^{3x+2} = 9^{2-x}\) (calculator) (4 marks)

**Solution:**

\[
\log[6(5)^{3x+2}] = \log 9^{2-x}
\]

\[
\log 6 + \log 5^{3x+2} = \log 9^{2-x}
\]

\[
\log 6 + (3x + 2) \log 5 = (2 - x) \log 9
\]

\[
\log 6 + 3x \log 5 + 2 \log 5 = 2 \log 9 - x \log 9
\]

\[
3x \log 5 + x \log 9 = 2 \log 9 - 2 \log 5 - \log 6
\]

\[
x(3 \log 5 + \log 9) = 2 \log 9 - 2 \log 5 - \log 6
\]

\[
x = \frac{2 \log 9 - 2 \log 5 - \log 6}{3 \log 5 + \log 9}
\]

\[
x = -0.088
\]
49. Evaluate: \( \log_4 2 \)  

Solution:  
\[
\frac{1}{2}
\]

50. Estimate the value of \( \log_2 5 \). Justify your answer.  

Solution:  
\[
\log_2 4 = 2 \\
\log_2 8 = 3 \\
\text{therefore } \log_2 5 \approx 2.3
\]

51. Sketch the graph of \( f(x) = 3 \log_2 (x + 1) \).  

[Graph of \( f(x) = 3 \log_2 (x + 1) \)]
52. Solve: \[ 4 \log_3 2 - \frac{1}{3} \log_3 8 = \log_3 a \] (3 marks)

Solution:
\[
\log_3 2^4 - \log_3 8^{\frac{1}{3}} = \log_3 a \\
\log_3 16 - \log_2 2 = \log_3 a \\
\log_3 \left( \frac{16}{2} \right) = \log_3 a \\
\log_3 8 = \log_3 a \\
a = 8
\]

53. Use the law of logarithms, fully expand the expression: \[ \log_a \left( \frac{x^3}{y\sqrt{z}} \right) \] (3 marks)

Solution:
\[
\log_a \left( \frac{x^3}{y\sqrt{z}} \right) = \log_a x^3 - \log_a y - \log_a z^{\frac{1}{2}} \\
= 3 \log_a x - \log_a y - \frac{1}{2} \log_a z
\]

54. Given \( f(x) = 2^x + 1 \), state the equation of the horizontal asymptote. (1 mark)

Solution:
\[ y = 1 \]
55. Sheeva’s bank is lending her $50 000 at an annual interest rate of 6%, compounded monthly, to purchase a car. Given that the last payment will be a partial payment, determine how many full monthly payments of $800 Sheeva will have to make. 

The formula below may be used.

\[ PV = \frac{R[1-(1+i)^{-n}]}{i} \]

where 
- \( PV \) = the present value of the amount borrowed
- \( R \) = the amount of each periodic payment
- \( i \) = \( \frac{\text{annual interest rate (as a decimal)}}{\text{the number of compounding periods per year}} \)
- \( n \) = the number of equal periodic payments.

Express your answer as a whole number. 

(3 marks)

**Solution**

\[ 50000 = \frac{800[1-(1+\frac{0.06}{12})^{-n}]}{\frac{0.06}{12}} \]

\[ 250 = 800[1-(1+0.005)^{-n}] \]

\[ 0.3125 = 1-(1+0.005)^{-n} \]

\[ 0.0675 = -1.005^{-n} \]

\[ 0.6875 = 1.005^{-n} \]

\[ \log 0.6875 = -n \log 1.005 \]

\[ \frac{\log 0.6875}{-\log 1.005} = n \]

\[ 75.12588088 = n \]

\[ \therefore \ 75 \text{ full monthly payments are needed} \]

3 marks
56. Using the laws of logarithms, fully expand the expression:  
\[
\log \left( \frac{w^3 x}{y - 1} \right)
\]

**Solution**

\[
3 \log_2 w + \log_2 x - \log_2 (y - 1)
\]

1 mark for power law  
1 mark for product law  
1 mark for quotient law

3 marks

57. Solve the following equation:  
\[
\log_3 (x + 3) + \log_3 (x - 5) = 2
\]

**Solution**

\[
\log_3 [(x + 3)(x - 5)] = 2
\]

1 mark for product law  
1 mark for exponential form

\[
(x + 3)(x - 5) = 3^2
\]

\[
x^2 - 2x - 15 = 9
\]

\[
x^2 - 2x - 24 = 0
\]

\[
(x - 6)(x + 4) = 0
\]

\[
x = 6  \quad x = -4
\]

\(\frac{1}{2}\) mark for solving for \(x\)  
\(\frac{1}{2}\) mark for rejecting extraneous root

3 marks
58. Solve: \[9^{2x+1} = 27^x\] (3 marks)

**Solution**

\[3^{2(2x+1)} = 3^{3x}\]

1 mark for changing to a common base

\[3^{4x+2} = 3^{3x}\]

1 mark for exponent law (½ mark for each side)

\[4x + 2 = 3x\]

½ mark for equating exponents

\[x = -2\]

½ mark for solving for \(x\)

3 marks

59. Expand using the laws of logarithms. (2 marks)

\[\log\left(\frac{a}{b^4}\right)\]

**Solution**

\[\log a - \log b^4\]

1 mark for quotient law

\[\log a - 4 \log b\]

1 mark for power law

2 marks
60. Peter invests $560 per month at an annual interest rate of 4.2%, compounded monthly. Determine how many monthly investments he will need to make to obtain at least $500 000. Express your answer as a whole number. (calculator) (3 marks)

Use the formula:

\[ FV = \frac{R[(1 + i)^n - 1]}{i} \]

where \( FV \) = the future value
\( R \) = the investment amount each period
\( i \) = the annual amount each period
\( n \) = the number of compounding periods per year
\( n \) = the number of investments

Solution

\[
500 \, 000 = \frac{560 \left[ \left(1 + \frac{0.042}{12} \right)^n - 1 \right]}{\frac{0.042}{12}}
\]

\[
500 \, 000 = \frac{560 \left(1 + 0.0035 \right)^n - 1}{0.0035}
\]

\[
500 \, 000 = 160 \, 000 \left(1.0035^n - 1\right)
\]

\[
3.125 = 1.0035^n - 1
\]

\[
4.125 = 1.0035^n
\]

\[
\log 4.125 = \log 1.0035^n
\]

\[
\log 4.125 = n \log 1.0035
\]

\[
n = \frac{\log 4.125}{\log 1.0035}
\]

\[
n = 405.584
\]

\[\therefore\] 406 monthly investments are needed.
61. Solve the following equation algebraically: 

\[
\log\left(\frac{x^2 + 5}{x^2 + 1}\right) = \log 3
\]

**Solution**

\[
\log\left(\frac{x^2 + 5}{x^2 + 1}\right) = \log 3
\]

1 mark for quotient law

\[
\frac{x^2 + 5}{x^2 + 1} = 3
\]

½ mark for equating arguments

\[
x^2 + 5 = 3(x^2 + 1)
\]

\[
x^2 + 5 = 3x^2 + 3
\]

\[
x^2 = 2
\]

\[
x = \pm1
\]

½ mark for solving for \(x\)

62. Justify why 4.7 is a better estimate than 4.3 for the value of \(\log_2 26\).

**Solution**

\[
2^4 = 16 \quad 2^5 = 32
\]

or

\[
\log_2 16 = 4 \quad \log_2 32 = 5
\]

26 is closer to 32 than 16; therefore \(\log_2 26\) is closer to 5 than 4.
63. Sketch the graph of \( y = -2^x + 2 \). (3 marks)

Solution

![Graph of \( y = -2^x + 2 \)](image)

1 mark for shape of an exponential function
1 mark for vertical reflection
1 mark for asymptotic behaviour approaching \( y = 2 \)

64. Explain why the domain of \( y = \log_2 (x - 1) \) is \( x > 1 \). (1 mark)

Solution

The argument of a logarithmic function must be positive. 1 mark
65. If \( \log 6 = p \), \( \log 5 = r \) and \( \log 2 = q \), express \( \log 60 \) in terms of \( p \), \( q \), and \( r \).

(2 marks)

**Solution**

\[
\log 60 = \log(6 \cdot 5 \cdot 2) \\
= \log 6 + \log 5 + \log 2 \\
= p + r + q
\]